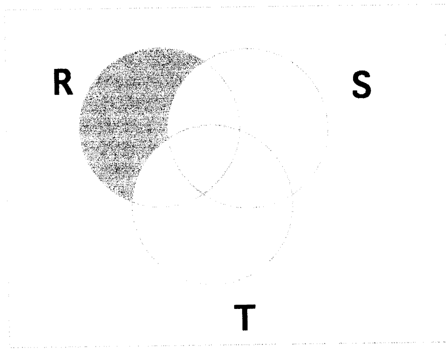
FALL 2006 FINAL

All Questions, 5 Points Each

1 Which of the following sets is represented by the shaded region in the Venn diagram below?



(a) $R \cap T$ (b) $R \cap (S \cup T)$ (c) $R \cup (S \cap T)$ (d) $R \cap (S \cup T)'$ (e) $R \cup S \cup T$

The shaded Region is in R but outside $S \cup T$

$R \cap (S \cup T)'$

2 If S and T are sets such that $n(S \cup T) = 20$, $n(S) = 15$, $n(S \cap T) = 3$, find $n(T)$.

(a) 10 (b) 8 (c) 15 (d) 2 (e) 5

IN-EX Principle:

$n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$20 = 15 + n(T) - 3$

$20 - 15 + 3 = n(T)$

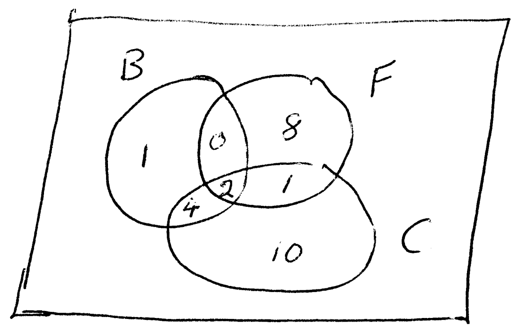
$8 = n(T)$

3 Out of 30 job applicants, 11 are female, 17 are college graduates, 7 are bilingual, 3 are female college graduates, 2 are bilingual women, 6 are bilingual college graduates, and 2 are bilingual female college graduates. The number of male college graduates is: $n(F' \cap C)$

(a) 18 (b) 14 (c) 4 (d) 10 (e) 15

B = Bilingual
F = Female
C = College Graduates

$n(F)$	11
$n(C)$	17
$n(B)$	7
$n(F \cap C)$	3
$n(F \cap B)$	2
$n(B \cap C)$	6
$n(B \cap F \cap C)$	2



$n(F' \cap C) = 4 + 10 = 14$

4 On the dinner menu at *The Hogs Breath Cafe*, there is a choice of 5 different appetizers, 10 main courses and 6 desserts. If you wish to order one appetizer, one main course and one dessert, how many different orders could you place?

(*a*) 900 (*b*) 21 (*c*) 300 (*d*) 210 (*e*) 400

	App	Main Course	Desert	
# ways	5 ×	10 ×	6	= 300

5 In a round robin Soccer tournament, each team will play each other team exactly once. If there are 10 teams entered in the tournament, how many games must be played?

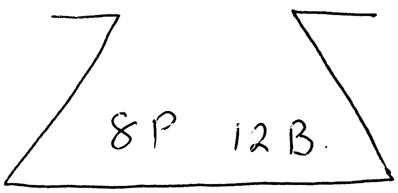
(*a*) 13 (*b*) 55 (*c*) 120 (*d*) 17 (*e*). 45

$C(10,2) = \frac{10 \cdot 9}{2 \cdot 1} = 45$ games

6 An urn contains 8 pink and 12 blue balls. A sample of size five is drawn from the urn. How many such samples have at least one pink ball.

(*a*) $C(20,5) - C(8,0)C(12,5)$ (*b*) $C(8,0)C(12,5)$ (*c*) $C(12,4)C(8,1)$

(*d*) $C(8,1) + C(8,2) + C(8,3) + C(8,4) + C(8,5)$ (*e*) $C(20,5) - C(8,0)$



Total = 20
Sample size = 5.

AT Least 1 Pink = complement of "ALL Blue"

Samples with at least one Pink

= Total # Samples − # samples with "all Blue"

$= C(20,5) - C(8,0)\,C(12,5)$

↑ 0 P ↑ 5 B.

7 How many four letter words, including nonsense words, can you make using the letters of the word

HOLIDAY

if each word must end with a vowel and letters cannot be repeated. (Note: Y is not a vowel)

(a) $C(7,4)$ (b) $P(7,4)$ (c) $5 \cdot 6 \cdot 7 \cdot 3$ (d) $4 \cdot 5 \cdot 6 \cdot 3$ (e) $2 \cdot 3 \cdot 4 \cdot 3$

We have 7 letters all of which are different

We place the vowel at the end first - we have 3 ways of completing this step.

$$4 \times 5 \times 6 \times 3$$

Since we cannot repeat letters, we have $6 \times 5 \times 4$ ways of filling in the other 3 letters.

8 A deck of cards consists of 52 playing cards, with four suits, diamonds, hearts, clubs and spades. Each suit has 13 cards in it. How many poker hands consist of four hearts and a card of a different suit?

(a) $48 \cdot C(13,4)$ (b) $39 \cdot C(52,4)$ (c) $48 \cdot C(52,4)$

(d) $39 \cdot C(13,4)$ (e) $C(13,4)C(13,1)$

13H 39 OTHERS

Sample Size 5

To choose a hand with 4 hearts and a card from a different suit

Step 1 : Choose 4 hearts $C(13,4)$ ways

Step 2 : Choose one other card. 39 ways

such hands $= C(13,4) \cdot 39$

9 Determine the coefficient of x^4y^7 in the binomial expansion of $(x+y)^{11}$.

~~(a)~~ $C(11,4)$ (b) 1 (c) $P(11,4)$ (d) 4 (e) $C(11,4)C(11,7)$

$(x+y)^{11} = C(11,0)x^0y^{11} + C(11,1)x^1y^{10} + \cdots + C(11,k)x^ky^{11-k} + \cdots + C(11,11)x^{11}$

x^4y^7 has coeff $C(11,k)$ with $k=4$

Coefficient $= C(11,4)$

10 An experiment consists of tossing six coins and counting **the number of heads**. The sample space for this experiment is $\{0,1,2,3,4,5,6\}$. If E is the event that there are more heads than tails, which of the following sets corresponds to E?

(a) $\{1,2,3,4,5,6\}$ ~~(b)~~ $\{4,5,6\}$ (c) $\{1,2,3\}$

(d) $\{3,4,5,6\}$ (e) $\{0,2,4,6\}$

More H's than T's means # H's > # T's.

# H's	# T's
0	6
1	5
2	4
3	3
4 >	2
5 >	1
6 >	0

$E = \{4, 5, 6\}$

11 If E and F are events in a sample space with probabilities

$$P(E) = .5, \qquad P(F) = .3.$$

If we know that E and F are independent events, What is $P(E \cup F)$?

(a) -0.2 (b) 0.2 (c) 0.8 (d) 0.15 ~~(e)~~ 0.65

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= .5 + .3 - P(E \cap F)$

Because E and F are independent $P(E \cap F) = P(E)P(F)$

$= (.5)(.3) = .15$

$P(E \cup F) = .5 + .3 - .15 = .8 - .15 = .65$

12 If you roll a pair of fair dice, one red and one green, what is the probability that the sum of the two numbers on the uppermost faces is 7?

(a) $\frac{7}{36}$ (b) $\frac{1}{36}$ ~~(c)~~ $\frac{1}{6}$ (d) $\frac{5}{36}$ (e) 0

The sample space has 36 equally likely outcomes

$\{(1,1)\ (1,2)\ \ldots\ (1,6)$

$(2,1)\ (2,2)\ \ldots\ (2,6)$

$\vdots \qquad \vdots$

$(6,1)\ (6,2)\ \ldots\ (6,6)\}$

The event "The sum of both numbers = 7"

is $A = \{(6,1)\ (5,2)\ (4,3)\ (3,4)\ (2,5)\ (1,6)\}$.

$P_r(A) = \frac{\# A}{\# S.S.} = \frac{6}{36} = \frac{1}{6}.$

13 An urn contains 5 pink and 6 blue marbles. A sample of 4 marbles is drawn from the urn. What is the probability that the sample has exactly 2 pink marbles?

(a) $\frac{C(5,\ 2)}{C(11,\ 4)}$ (b) $\frac{1}{C(11,\ 4)}$ ~~(c)~~ $\frac{C(5,\ 2)C(6,\ 2)}{C(11,\ 4)}$ (d) $\frac{2}{11}$ (e) $\frac{1}{2}$

5P 6B

Sample size 4.

$Pr(\text{Exactly 2 Pink}) = \frac{\#\text{ Samples WITH Exactly 2P}}{\text{TOTAL }\#\text{ Samples}}$

$= \frac{\#\text{ Samples WITH 2P and 2B}}{C(11,4)} = \frac{C(5,2)\,C(6,2)}{C(11,4)}$

14 The rules of a carnival game are as follows:

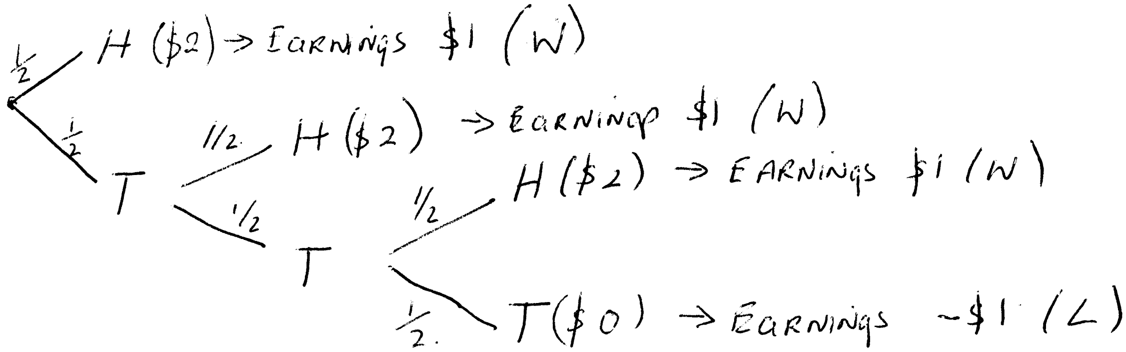
You pay \$1 to play

You then flip a coin until you get a head or have flipped the coin three times.

If you get a head, you win and the carnival attendant gives you \$2. If you don't get a head, you lose, the carnival attendant gives you nothing.

What is the probability that you win the game? (A tree diagram might help)

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ ~~(d)~~ $\frac{7}{8}$ (e) $\frac{3}{4}$



$Pr(W) = \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}.$

15 The Everlasting Lightbulb company produces lightbulbs, which are packaged in boxes of 20 for shipment. Tests have shown that 5% of the lightbulbs produced in Everlasting Lightbulb factory are defective. What is the probability that a box, ready for shipment, contains exactly 3 defective lightbulbs?

(*a*) $1 - C(20,3)(.05)^3(.95)^{17}$

(*b*) $(.95)^{20} + C(20,1)(.05)^1(.95)^{19} + C(20,2)(.05)^2(.95)^{18}$

(*c*) $1 - \left\{ (.95)^{20} + C(20,1)(.05)^1(.95)^{19} + C(20,2)(.05)^2(.95)^{18} \right\}$

X (*d*) $C(20,3)(.05)^3(.95)^{17}$

(*e*) $1 - (.95)^{20}$

Q: In 20 repeats of the experiment: "choose a Lightbulb from those produced @ the Everlasting Lightbulb factory" ~~What~~ Let X = # Defective Lightbulbs. What is Pr(X=3)?
X is a binomial Random Variable with $n = 20$ $p = .05$ $q = .95$

$Pr(X = 3) = C(n, k) p^k q^{n-k} = C(20, 3)(.05)^3(.95)^{17}$

16 Samantha records her times for 10 crosscountry 5 mile races with the following results (given in minutes):

$$\frac{25.8 + 25.5 + 25.8 + 25.4 + 25.6 + 25.9 + 25.7 + 25.6 + 25.9 + 25.7}{10} = 25.69$$

What is Samantha's average running time per race, for those 10 races.

X (*a*) 25.69 (*b*) 25.75 (*c*) 25.9 (*d*) 25.82 (*e*) 25.3

17 Find the population variance, $\sigma^2 = Var(X)$, of the random variable X, whose distribution is given in the table below.

Outcome	Probability
0	.1
1	.2
2	.4
3	.2
4	.1

(a) 1 (b) 1.2 (c) 8 (d) 0 (e) .5

18 The number of cups of Starbucks coffee consumed by Netty the Caffiene addict each week is normally distributed with mean $\mu = 15$ and standard deviation $\sigma = 2$. Each cup of coffee costs \$1.50. What is the probability that Netty spends \$30 or more on coffee this week.

(a) .0287 (b) .0115 (c) .9938 (d) .9713 (e) .0062

19 If Z is a standard normal random variable, with mean $\mu = 0$ and standard deviation $\sigma = 1$, what is

$$Pr(-1.5 \leq Z \leq 2)?$$

(a) .9772 (b) .0668 (c) .9104 (d) .044 (e) .0896

20 Consider the matrices

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix}_{2\times 3} \quad \text{and} \quad B = \begin{pmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 3 \end{pmatrix}_{3\times 2}.$$

Which of the following gives the product of the two matrices, $A \cdot B$?.

(a) $\begin{pmatrix} -4 & 2 & 0 \\ 2 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} -2 & 0 & 0 \\ 4 & -1 & 1 \\ 0 & 1 & 5 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 5 \\ 5 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -2 & 2 \\ 4 & -2 \end{pmatrix}$

(e) It is not possible to form the product of these two matrices.

$(AB)_{2\times 2}$ → must be (c) or (d)

$$\overset{A}{\begin{pmatrix} -1 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix}} \overset{B}{\begin{pmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 3 \end{pmatrix}} = \begin{pmatrix} -2 & 2 \\ 4 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} (-1)\cdot 2 + (1)0 + 0(1) & (-1)(-1) + (1)(1) + 0(3) \\ 2(2) + 0(0) + 0(1) & 2(-1) + 0(1) + 0(3) \end{pmatrix}$$

21 Find the inverse of the following matrix:

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$A = \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$$

A^{-1} exists if $a \cdot d - bc \neq 0$

$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(a) $\begin{pmatrix} 0 & -1 \\ -2 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & -1/2 \\ -1 & -1/2 \end{pmatrix}$ ~~(c)~~ $\begin{pmatrix} 0 & 1/2 \\ 1 & 1/2 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1/2 \\ 1 & -1/2 \end{pmatrix}$ (e) $\begin{pmatrix} 0 & -1 \\ -2 & 1 \end{pmatrix}$

Inverse exists if $(-1)(0) - (2)(1) \neq 0$ True so A^{-1} exists

$= -2$

$A^{-1} = \frac{1}{-2}\begin{pmatrix} 0 & -1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ 1 & \frac{1}{2} \end{pmatrix}$

check: $\begin{pmatrix} 0 & \frac{1}{2} \\ 1 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

22 Sunbucks coffee shop sells two types of caffeinated coffee blends: traditional dark roast and mild breakfast blend. Each pound of traditional dark roast contains 80% Jamaican bean and 20% Alaskan bean, and earns a profit of \$6 per pound. Each pound of mild breakfast blend contains 35% Jamaican bean and 65% Alaskan bean, and earns a profit of \$5 a pound. Sunbucks has 3000 pounds of Jamaican bean and 2500 pounds of Alaskan bean. If x is the number of pounds of the traditional dark roast produced and y is the number of pounds of the mild breakfast blend produced, then the inequalities that must be saitisfied for Sunrbucks to maximize its profit are :

(a)
$.80x + .20y \leq 3000$
$.35x + .65y \leq 2500$
$x \geq 0, y \geq 0$
profit: $6x + 5y$

(b)
$.80x + .35y \leq 3000$
$.65x + .20y \leq 2500$
$x \geq 0, y \geq 0$
profit: $5x + 6y$

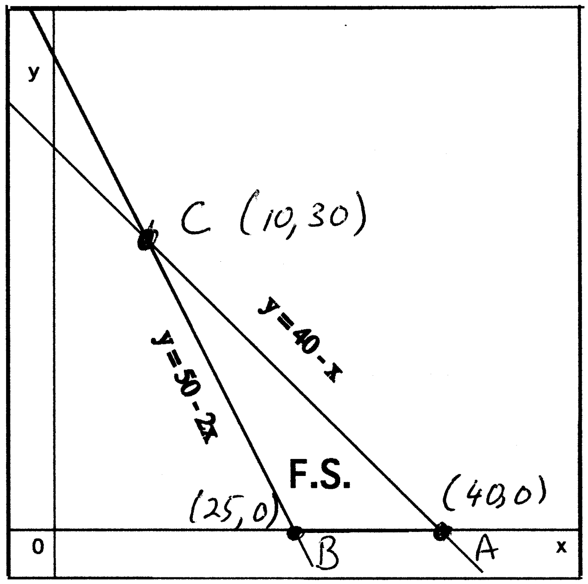
(c)
$.80x + .20y \geq 3000$
$.35x + .65y \geq 2500$
$x \geq 0, y \geq 0$
profit: $6x + 5y$

(d)
$.35x + .80y \geq 3000$
$.65x + .20y \geq 2500$
$x \geq 0, y \geq 0$
profit: $5x + 6y$

~~(e)~~
$.80x + .35y \leq 3000$
$.20x + .65y \leq 2500$
$x \geq 0, y \geq 0$
profit: $6x + 5y$

	x T.D.R	y M.B.B.		
Jam. Bean	.8	.35	3000	$.8x + .35y \leq 3000$
Al. Bean	.2	.65	2500	$.2x + .65y \leq 2500$
Profit	\$6	\$5		$6x + 5y$

23 Find the maximum of the objective function, $3x + 4y$ on the feasible set drawn below.



(*a*) 40 (*b*) 150 (*c*) 120 (*d*) 75 (*e*) 200

24 Consider the points

$$D : x = 30, \quad y = 15$$
$$E : x = 25, \quad y = 20.$$
$$F : x = 35, \quad y = 2$$

Using the feasible set given in Question 23, Which of the following statements are true?

(a) F is in the feasible set

(b) E and F are in the feasible set

(c) D and E are in the feasible set

(d) E is in the feasible set

(e) D and F are in the feasible set

25 Roadrunner (R) and Coyote (C) play a zero-sum game, with the pay-off matrix for Roadrunner, R, given by :

	C_1	C_2
R_1	1	3
R_2	5	2

In order to solve for his optimal mixed strategy, which of the Linear Programming problems given below must Roadrunner solve?

(*a*) maximize $x+y$ subject to constraints $x \geq 0, \quad y \geq 0$; $x+5y \leq 1$; $3x+2y \leq 1$

(*b*) minimize $x+y$ subject to constraints $x \geq 0, \quad y \geq 0$; $x+3y \leq 1$; $5x+2y \leq 1$

(*c*) minimize $x+y$ subject to constraints $x \geq 0, \quad y \geq 0$; $x+3y \geq 1$; $5x+2y \geq 1$

(*d*) maximize $x+y$ subject to constraints $x \geq 0, \quad y \geq 0$; $2x+y \geq 1$; $x+3y \geq 1$

(*e*) minimize $x+y$ subject to constraints $x \geq 0, \quad y \geq 0$; $x+5y \geq 1$; $3x+2y \geq 1$

MINIMIZE $x+y$. subject to

$$(x \quad y)\begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix} \geq (1,1) \Rightarrow (x+5y \quad 3x+2y) \geq (1,1)$$

$x \geq 0, y \geq 0$. or $x+5y \geq 1$, $3x+2y \geq 1$

26 Rachmaninof (R) and Chaicovsky (C) play zero-sum game, where Rachmaninof's pay-off matrix is given by:

	C_1	C_2	C_3	C_4	C_5	min
R_1	2	1	−1	3	−2	−2
R_2	4	2	−2	0	5	−2
R_3	3	4	1	−3	−1	−1
→ R_4	0	2	1	3	2	0
R_5	3	2	1	4	−1	−1

What is Rachmaninof's optimal pure (fixed) strategy for this game?

(*a*) Row 1 (*b*) Row 2 (*c*) Row 3 (*d*) Row 4 (*e*) Row 5

27 Rebecca and Consuela play a zero sum game, where the pay off matrix for Rebecca is given by:

	C_1	C_2	C_3
R_1	4	2	−2
R_2	3	4	1
R_3	2	1	−1

If the payoff matrix has a saddle point where is it?

(*a*) Row 3, Col 3 (*b*) Row 1, Col 3 (*c*) Row 2, Col 3 (*d*) Row 3, Col 1 (*e*) Row 2, Col 2

28 Rudolph (R) and Comet (C) play a game. They both choose a number between 1 and 4 simultaneously. Comet gives Rudolph a number of carrots equal to the sum of the two numbers chosen minus three. If this number is negative, Comet recieves carrots from Rudolph. Which of the following give the pay-off matrix for Rudolph?

(*a*)

	No.	*C* 1	2	3	4
R	1	−1	0	1	2
	2	0	1	2	3
	3	1	2	3	4
	4	2	3	4	5

(*b*)

	No.	*C* 1	2	3	4
R	1	3	2	1	2
	2	2	1	2	3
	3	1	2	3	4
	4	2	3	4	5

(*c*)

	No.	*C* 1	2	3	4
R	1	2	3	4	5
	2	1	0	−1	−2
	3	0	1	2	3
	4	−1	0	1	2

(*d*)

	No.	*C* 1	2	3	4
R	1	−1	0	2	2
	2	0	0	2	0
	3	1	2	3	3
	4	2	3	4	4

(*e*)

	No.	*C* 1	2	3	4
R	1	3	4	5	6
	2	4	5	6	7
	3	5	6	7	8
	4	6	7	8	9

29 Rasputin (R) and Catherine (C) play a zero-sum game with payoff matrix for Rasputin given below. If Rasputin's strategy is given by (.3, .2, .5) and Catherine's strategy is given by $\begin{pmatrix} .2 \\ .1 \\ .7 \end{pmatrix}$, what is the expected pay-off for Rasputin?

	C_1	C_2	C_3
R_1	1	0	0
R_2	1	5	0
R_3	3	2	−2

~~(a)~~ −0.1 (b) 1 (c) 1.54 (d) −0.3 (e) 0.21

Expected Pay-off for Rasputin:

$$(.3 \;\; .2 \;\; .5)\begin{pmatrix} 1 & 0 & 0 \\ 1 & 5 & 0 \\ 3 & 2 & -2 \end{pmatrix}\begin{pmatrix} .2 \\ .1 \\ .7 \end{pmatrix} = \left(\underset{2}{.3+.2+1.5} \;\; \underset{2}{1+1} \;\; \underset{-1}{-1}\right)\begin{pmatrix} .2 \\ .1 \\ .7 \end{pmatrix}$$

$$= (.4+.2-.7) = -.1$$

30 Cop (C) and Robber (R) play a zero-sum game, with payoff matrix for Robber given by

	C_1	C_2
R_1	2	1
R_2	1	3

If the solution to the linear programming problem:

$$\begin{array}{ll} \text{minimize} & x+y \\ \text{constraints} & x \geq 0, \quad y \geq 0 \\ & 2x+y \quad \geq 1 \\ & x+3y \quad \geq 1 \end{array}$$

is given by $x = \frac{2}{5}$, $y = \frac{1}{5}$, which of the following give the optimal strategy and ν = expected payoff for Robber?

(a) $\left(\frac{1}{3} \; \frac{2}{3}\right)$, $\nu = \frac{5}{3}$ (b) $\left(\frac{6}{25}, \frac{3}{25}\right)$, $\nu = \frac{3}{5}$ (c) $\left(\frac{2}{3}, \frac{1}{3}\right)$, $\nu = \frac{3}{5}$

(d) $\left(\frac{2}{5}, \frac{1}{5}\right)$, $\nu = \frac{3}{5}$ ~~(e)~~ $\left(\frac{2}{3}, \frac{1}{3}\right)$, $\nu = \frac{5}{3}$

Optimal strategy $= (r_1 \;\; r_2) = \left(\frac{x}{x+y} \;\; \frac{y}{x+y}\right) = \left(\frac{\frac{2}{5}}{\frac{2}{5}+\frac{1}{5}} \;\; \frac{\frac{1}{5}}{\frac{2}{5}+\frac{1}{5}}\right)$

$= \left(\frac{2}{3} \;\; \frac{1}{3}\right)$ (r_1, r_2)

$\nu = \frac{1}{x+y} = \frac{1}{\frac{2}{5}+\frac{1}{5}} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$